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A Thermodynamic Approach to the Enhancement of Reliability and the Optimization of Energy Efficiency of Centrifugal Compressors

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Abstract

The Aerothermodynamic performance of centrifugal compressors is substantially influenced by the variation in suction parameters and distortion in the internal clearances or the gas flow profile. Furthermore, deposits buildup can also affect the rotordynamic response of the system which can result in excessive vibration amplitudes due to the change in the synchronous dynamic forces acting on the rotor-bearing system. Therefore, thermodynamic performance analysis is an effective diagnostic tool for optimizing energy consumption and the reliability of the compressor through early prediction of machine failure and efficiency deterioration. This work introduces an approach to modeling and monitoring of the thermodynamic performance of centrifugal compressors. The approach relies on GERG-2008 equation of state for calculating gas properties at suction and discharge states. Additionally, Huntington 4-point method is used for evaluating the polytropic efficiency and polytropic head. A computer algorithm has been developed to Calculate the thermal and caloric properties of the gas at the compressor suction and discharge using GERG-2008 equation of state. In addition, it evaluates the polytropic head and efficiency using Huntington 4-Point Method. The program was also designed to Generate corrected performance maps based on the existing suction conditions and the manufacturer as-tested maps. The code can be run on historical data and produce trend plots for polytropic efficiency with temperature, pressure, flow rate, and speed. The calculated gas properties and polytropic efficiency were validated against literature results. Moreover, the developed algorithm was tested on several machines to assess their efficiency and energy consumption over different periods of time. The generated trends can be utilized to diagnose the root cause of efficiency degradation. Overall, the developed approach provides accurate and instantaneous results that can assist the compressor end-users in making critical decisions regarding the optimum time to halt operation of the highly demanding machines for carrying out major maintenance activities.

Keywords: centrifugal compressor, polytropic efficiency, compressor reliability, gas properties, equation of state

Introduction

Centrifugal compressors are essential components in the oil and gas industry, playing vital roles in various processes across the entire industry, from upstream operations to downstream processes. The failure of these major pieces of equipment can result in a complete unit or plant unplanned shutdown, leading to prolonged downtime and production deferment. Therefore, continuous online health monitoring is necessary to ensure enhanced reliability, cost-effectiveness, high production rates and reduced Mean Time Between Failures (MTBF).

One method of monitoring the health of centrifugal compressors involves evaluating the vibration response of the machine. This approach proves highly efficient in early and precise detection of mechanical problems such as unbalance, misalignment, and looseness, ensuring effective diagnosis of such malfunctions. Moreover, vibration analysis can occasionally offer insights into aerodynamic deterioration, as the accumulation of non-reactive deposits on the rotor can shift the residual unbalance force, resulting in gradual changes in the synchronous vibration amplitude and phase angle. Nevertheless, it is important to note that changes in synchronous dynamic response can sometimes be a lagging indicator, particularly at low speeds. This is because the non-reactive deposit mass sticking to the rotor must be sufficiently high to produce a noticeable vibration change. Furthermore, foreign materials can adhere to the stationary components of the gas flow path, having minimal impact on the rotor dynamic response.

Apart from material accumulation, there are various factors that can contribute to the degradation of the compressor aerothermodynamic performance. These factors can be classified into three main categories as outlined below (Al Busaidi and Pilidis 2016):

- Variation in the suction parameters which include fluctuations in the suction pressure, temperature, and the properties of the gas mixture.
- Distorted gas flow path caused by problems such as fouling or recirculation that is attributed to excessive clearances resulting from seal failures.
- Damage in the compressor internal components such as impeller and diffuser.

To achieve more accurate diagnosis of performance degradation resulting from the aforementioned factors, many users of compressors rely on thermodynamic performance analysis. This analysis involves calculating the polytropic head, polytropic efficiency, and comparing the current performance with the performance obtained during the performance test conducted by the original equipment manufacturer (OEM) when the compressor was aerodynamically sound. Furthermore, accurate calculation of the polytropic efficiency can help in evaluating and optimizing the energy consumption and reduce the carbon emissions caused by operating these energy intensive machines away from the best efficiency point (BEP).

In general, evaluating the aerothermodynamics performance consist of the following two steps.

1. Compute the thermodynamic properties of the gas mixture at the suction and discharge states.
2. Use the calculated gas properties to work out the polytropic head and efficiency.

To ensure more accurate performance evaluation, it is essential to compare the calculated head and efficiency with the reference values obtained from a performance test conducted when the compressor was in optimal condition. It is important to note that if the current inlet conditions significantly differ from the reference conditions, adjustments need to be made to correct the reference head and efficiency values.

A significant number of end-user companies worldwide employ primitive methods for calculating the polytropic head and efficiency. These methods heavily rely on ideal gas assumptions, with limited consideration for the advanced theories of real gases. Adopting such simplification can be attributed to the limited availability of comprehensive packages capable of integrating real gas thermodynamics with compressor aerodynamics, combining both theories to achieve accurate performance evaluation.

Furthermore, the calculation of thermodynamic gas properties and the evaluation of aerodynamic performance based on modern theories require significant computational resources, making it infeasible to perform such calculations using simple software.

The objective of this study is to present an approach that utilizes the latest theoretical methods for assessing the aerothermodynamic performance of centrifugal compressors. To implement this approach, a computer algorithm was developed using MATLAB®. The obtained results are then compared with the most commonly used methods employed by end-users for calculating the polytropic head and efficiency.

Equations of State

The initial step in evaluating the compressor performance is to calculate the thermodynamic properties of the gas at the suction and discharge states. The specific properties needed for this calculation may vary depending on the chosen method. In order to determine the gas properties, two intensive and independent properties are initially required. In the case of centrifugal compressor applications, the two available properties for direct measurement are pressure and temperature. The remaining properties are subsequently calculated using an equation of state (EoS).

The first EoS ever introduced is the ideal gas equation of state which is represented by equation)1(. Since then, many real gas equations of state have been developed to account for deviations from ideal gas behavior. The first real gas equation of state was introduced by (Waals 1873). The general form of Van der Waals EoS is given by equation)2(, where the coefficients ***a*** and ***b*** can be obtained by equations)3(and)4(respectively.

$$Pv = RT \tag{1}$$

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \tag{2}$$

$$a = \frac{27R^2T_c^2}{64P_c} \tag{3}$$

$$b = \frac{RT_c}{8P_c} \tag{4}$$

(Lüdtke 2004) provided an overview of four frequently employed equations of state within the oil and gas industry: RKS, Peng Robinson, BWRS, and LKP. The four equations are summarized in Table 1. All four equations rely on the concept of corresponding states as their underlying principle. Peng Robinson and RKS equations of state are categorized as cubic equations of state, whereas BWRS and LKP belong to the virial-type equations of state. Cubic equations of state are P-v-T relations that are implicit in the molar volume with the highest power equals three which makes them easy to be solved analytically. On the other hand, while virial equations of state are generally more accurate, they are also more complex compared to cubic equations and require greater computational power. This is because their solution involves iterative numerical methods. Out of the four equations mentioned, LKP (Lee-Kesler-Plöcker) is regarded as the most accurate equation for hydrocarbon applications, with a maximum error of 0.7 % (Lüdtke 2004). (Plöcker 1977) expanded upon the LKP solution by incorporating enthalpy and heat capacities calculations. Furthermore, (Lüdtke 2004) presented a solution for determining the polytropic and isentropic exponents related to volume and temperature. Although these equations offer accurate solutions, they rely on the availability of experimentally determined binary interaction coefficients to solve for real gas mixtures. However, these coefficients are often not readily accessible to end-users for all components in the gas mixture. Considering the practicality for end-users, alternative options were explored in this work.

Table 1—Four of the Most Commonly Used EoS fore Centrifugal Compressors Applications

Equation	General Form	Equation Type	First Publishment
RKS	$Z=\frac{v}{(v-b)}-\frac{a}{RT(v+b)}$	Cubic	(Kowng and Redlich 1949)
Peng Robinson	$Z=\frac{v}{(v-b)}-\frac{\left(\frac{v}{RT}\right)a}{[v(v+b)+v(v-b)]}$	Cubic	(Peng and Robinson 1976)
BWRS	$Z=1+\frac{B}{v}+\frac{c}{v2}+\frac{D}{c5}+\left(\frac{\varsigma}{v2}\right)\left(1+\frac{\gamma}{v2}\right)e^{\frac{-\gamma}{v2}}$	Virial	(Benedict, et al. 1940)
LKP	$Z=Z^{(0)}+\left(\frac{\omega}{\omega(r)}\right)\left(Z^{(r)}-Z^{(0)}\right)$	Virial	(Lee and Kesler 1975)

AGA-8 and GERG-2008 are two of the most advanced equations of state frequently utilized in the oil and gas sector and adopted by ISO-20765. In comparison to the four equations examined by (Lüdtke 2004), these two equations are significantly more intricate and sophisticated. AGA-8, developed by (Starling and Savidge 1992) and published by the American Gas Association, is an equation of state (EoS) specifically tailored for gas-phase applications. It is characterized by its explicit representation of the compressibility factor and is commonly employed to determine the thermodynamic properties of natural gas mixtures composed of a maximum of 21 components. The temperature range in which AGA-8 is applicable is 143 K to 673 K, while it can accommodate pressures of up to 280 MPa.

GERG-2008, developed by (Wagner and Kunz 2012), is an extension of GERG-2004 initially introduced by (Kunz, et al. 2007). This equation of state (EoS) is explicitly solved for Helmholtz free energy and is capable of handling natural gas mixtures comprising up to 21 components. It is applicable across different phases, including the gas phase, liquid phase, supercritical region, and vapor-liquid equilibrium (VLE), for any combinations of these 21 components. The valid temperature range for GERG-2008 is 90 K to 450 K, and it can accurately handle pressures up to 35 MPa. The 21 pure substances considered in this equation are methane, nitrogen, carbon dioxide, ethane, propane, n-butane, isobutane, n-pentane, isopentane, n-hexane, n-heptane, n-octane, n-nonane, n-decane, hydrogen, oxygen, carbon monoxide, water, hydrogen sulfide, helium, and argon.

Despite being the most complex equation among the ones mentioned earlier, GERG-2008 equation of state (EoS) was chosen as the adopted model for this research due to its exceptional accuracy. Additionally, all the necessary coefficients required for its implementation are readily available in the open literature.

Compressor Head and Efficiency Calculation

Definition of Compression Efficiency

Compression efficiency serves as a gauge of the effectiveness of the compression process, measuring its overall quality (Robinson and Casey 2021). Typically, it is assessed by comparing the actual work input to an ideal work input that assumes no losses. Therefore, the compression efficiency can be defined as the ratio between the ideal work input and the actual work input.

$$Compression\ Effeciency=\frac{Ideal\ Work\ Input}{Actual\ Work\ Input}=\frac{y_{ideal}}{y_{actual}} \tag{5}$$

In general, the ideal work can be categorized as isentropic, polytropic, or isothermal. This classification depends on the thermodynamic quantity that remains constant throughout the reference compression process. It is worth mentioning that in the literature, the terms work and head are used interchangeably within this context.

The significant advantage of utilizing polytropic efficiency is its independence from preheating and pressure ratio, resulting in the total compressor head being equal to the sum of the individual stages' heads (Lüdtke 2004). Consequently, if two compressors are aerodynamically identical, their polytropic efficiency will remain the same irrespective of the pressure ratio. This makes polytropic efficiency a more practical tool for performance comparison when compared to isentropic efficiency. Considering all the aforementioned reasons, polytropic analysis will be employed as the preferred approach throughout this research.

Polytropic Head and Efficiency Models.

To find out the denominator of equation (5), the first law of thermodynamics is applied to the control volume. Solving the energy balance equation yields the conclusion that the actual work input is approximately equal to the total enthalpy difference between the compressor discharge and suction. This enthalpy difference can be calculated using an EoS. A large number of compressor users do not calculate the enthalpy difference due to the lack of software that can compute properties and combine the results with polytropic efficiency models to provide real time results. Instead, they rely on the actual work input from the driving machine. However, this approach is not accurate for evaluating compression effectiveness since it includes mechanical, aerodynamic, and electrical losses of the entire train. Similarly, other users rely on properties provided by the original equipment manufacturer (OEM) during the design phase, assuming a constant gas composition. Both methods can yield unreliable outcomes and potentially misguide maintenance decisions.

The polytropic head (work) is defined by equation (6):

$$y_p = \int_s^d v\, dP \tag{6}$$

Equation (7) is the prevailing model employed by compressor users to solve the integral mentioned above. However, this solution heavily relies on several ideal gas assumptions. The initial endeavor to model polytropic efficiency with minimal ideal gas assumptions was introduced by (Schultz 1962) who introduced the concepts of compressibility functions and polytropic head factor, which are utilized to adjust equation)7 (to approximate real gas behavior more accurately. (ASME PTC-10 1997) has adopted this method, which is extensively utilized by compressor manufacturers during performance testing.

$$y_p = Z_s R T_s \frac{n_v}{n_n - 1}\left[\left(\frac{P_d}{P_s}\right)^{\frac{n_v - 1}{n_v}} - 1\right] \tag{7}$$

Since 1962, several models have been developed to solve the integral mentioned above and offer more precise solutions for the polytropic head and efficiency. Broadly speaking, these methods can be classified into two categories: endpoint methods and stepped numerical methods. Endpoint methods involve calculating the integral using properties derived from the equation of state (EoS) for suction and discharge conditions. On the other hand, stepped numerical methods employ a more intensive and iterative process to evaluate the integral for the polytropic head. Table 2 provides a comprehensive list of the available methods. Two thorough studies were conducted by (Evans and Huble 2017), as well as by (Huntington 2017), where they extensively compared most of the existing methods. Generally, stepped numerical methods tend to offer greater accuracy. However, this enhanced accuracy comes at the expense of increased complexity and the need for significant computational power and time. In this study, the Huntington 4-Point method was chosen for its simplicity and outstanding accuracy, surpassing certain stepped numerical methods.

Table 2—Centrifugal Compressors Polytropic Efficiency Models and Years of Publishment

End Point Methods	Stepped Numerical Methods
Schultz (1962)	Huntington Reference (1985)
Mallen and Saville (1977)	Small Stage (2006)

End Point Methods	Stepped Numerical Methods
Sandberg and Colby (2013)	Sandberg and Colby Reference (2013)
Schultz XY (2&3 Point) (1962)	Nathoo and Gottenberg (1983)
Huntington 3 Point (1985)	Oldrich (2010)
Huntington 5 Point (2017)	Wettstein (2014)
Taher–Evans Cubic Polynomial (TE-CP) (2021)	Huntington 2017 Reference (2017)

Methodology

The evaluation approach proposed in this study is depicted in Figure 1. It incorporates a combination of three distinct models, namely:

- The GERG-2008 EoS, utilized for calculating thermodynamic gas properties.
- The Huntington 4-Point Method, employed to determine the polytropic head and efficiency.
- A similitude model, employed to correct the compressor performance map.

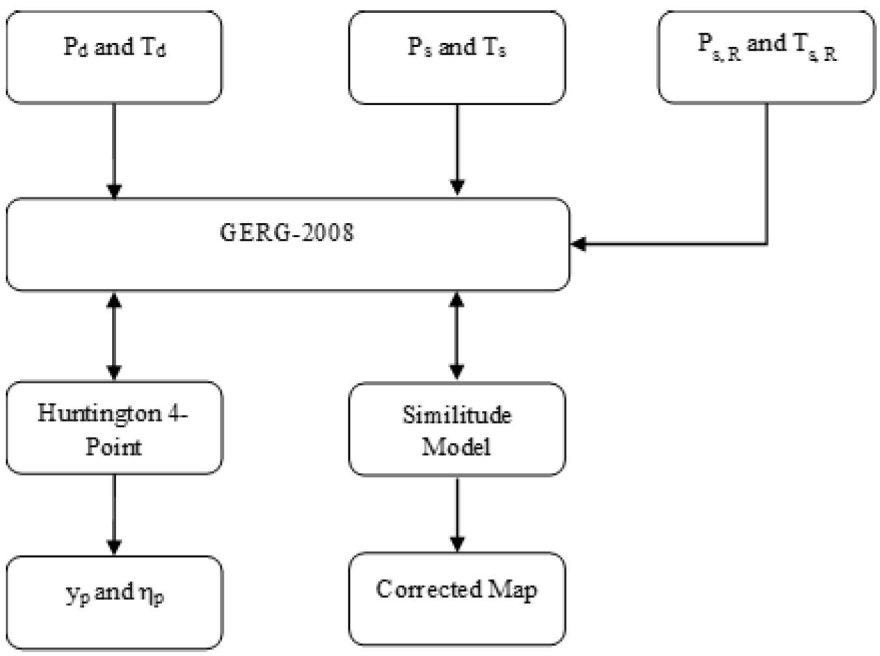

Figure 1—Performance Evaluation Flowchart

GERG-2008

The core foundation of the overall approach is based on the GERG-2008 equation of state. This equation receives pressure and temperature as input parameters at both the suction and discharge states during operating and design conditions. Subsequently, the equation provides the necessary properties required for further analysis to the Huntington 4-Point and Similitude models. GERG-2008 is explicitly solved for the Helmholtz free energy. Its general form is given by equation (8):

$$\alpha(\delta, \tau, x) = \alpha^{\circ}(\rho, \tau, x) + \alpha^{r}(\delta, \tau, x) \tag{8}$$

Following that, a computer program is developed to solve for both the ideal and residual components of the Helmholtz free energy. Moreover, the program calculates the derivatives of the Helmholtz energy with respect to the mole fraction and reduced mixture variables. Eventually, the necessary properties are obtained by utilizing their relations with the Helmholtz energy. The required relations are given in Table 3.

Table 3—Relations of the Required Properties with Helmholtz Free Energy

Property	Relation with Helmholtz Free Energy
Pressure	$\frac{P(\delta,\tau,x)}{\rho RT} = 1+\delta\alpha_{\delta}^{r}$
Compressibility Factor	$Z(\delta,\tau,x) = 1+\delta\alpha_{\delta}^{r}$
Entropy	$\frac{s(\delta,\tau,x)}{RT} = \tau(\alpha_{\tau}^{o}+\alpha_{\tau}^{r}) - \alpha^{o} - \alpha^{r}$
Enthalpy	$\frac{h(\delta,\tau,x)}{RT} = 1+\tau(\alpha_{\tau}^{o}+\alpha_{\tau}^{r})+\delta\alpha_{\delta}^{r}$
Sonic Speed	$\frac{w2(\delta,\tau,x)M}{RT} = 1+2\delta\alpha_{\delta}^{r}+\delta^{2}\alpha_{\delta\delta}^{r} - \frac{(1+\delta\alpha_{\delta}^{r}-\delta\tau\alpha_{\delta\tau}^{r})}{\tau 2(\alpha_{\tau\tau}^{o}+\alpha_{\tau\tau}^{o})}$

It is worth mentioning that GERG-2008 is specifically designed to accept molar density and temperature as inputs. However, in compressor applications, the parameters available for direct measurement are typically pressure and temperature. Therefore, a numerical method becomes necessary to determine the molar density corresponding to a given pressure and temperature. The secant method was used in this research.

Huntington 4-Point Method

The properties calculated using the GERG-2008 equation of state are subsequently inputted into the Huntington 4-Point model. This model utilizes the obtained properties to further analyze and predict the polytropic compression path. By incorporating the GERG-2008 properties into the Huntington 4-Point model, a comprehensive understanding of the gas compression process can be achieved, allowing for accurate assessment and optimization of the system's performance. Equation (9), proposed by (Huntington 2017) expresses the compressibility factor as a general function of pressure by taking four steps along the polytropic compression path.

$$Z = a+b+\left(\frac{P}{P_s}\right)+c\,ln\left(\frac{P}{P_s}\right)+d\left(\frac{P}{P_s}\right)ln\left(\frac{P}{P_s}\right) \tag{9}$$

The constants a, b, c, and d in equation (9) are determined by simultaneously solving the equation at the four states. In order to achieve this, the temperature at the two intermediate points needs to be estimated. Consequently, an iterative calculation process is necessary. (Huntington 2017) found that three iterations are adequate for accurate conversion, although he recommended five iterations for better accuracy. Finally, the polytropic efficiency is calculated using equation (10).

$$(s_d - s_s) = R\frac{(1-\eta)}{\eta}\left\{a.\,ln\frac{P_d}{P_s}+b.\left(\frac{P_d}{P_s}-1\right)+\frac{c}{2}\left(ln\frac{P_d}{P_s}\right)^2+d\left[\frac{P_d}{P_s}ln\frac{P_d}{P_s}-\left(\frac{P_d}{P_s}-1\right)\right]\right\} \tag{10}$$

Similitude Model

In order to improve the accuracy of evaluating the performance obtained from the Huntington 4-Point model, it is essential to compare the results with a reference performance. This reference performance acts as a benchmark or standard against which the actual performance of the compression process is evaluated. By conducting this comparative analysis, a more dependable evaluation of the system's performance can be achieved, leading to better maintenance optimization and potential enhancements in the efficiency and energy consumption.

The similitude concept assumes that a compressor will exhibit the same performance when operating at the same dimensionless flow, head, and Mach number (Aungier 2000). This concept suggests that compressors with similar flow coefficients and Mach numbers will experience comparable performance characteristics at different suction conditions. By employing the similitude concept, valuable predictions can be made for comparisons between actual and expected performance.

Equations (11), (12) and (13) are used to nondimensionalize the reference performance curve. The new performance curve is then derived by using the same equations under the new conditions.

$$\psi = \frac{2y_p}{U^2} \tag{11}$$

$$\phi = \frac{4Q}{\pi D^2 U} \tag{12}$$

$$Mu = \frac{U}{w} \tag{13}$$

Discussion of Results

The GERG-2008 model, which was developed, was compared to the NIST Reference Fluid Thermodynamic and Transport Properties Database (REFPROP). When examining pure substances, the calculated properties matched exactly between the two models. However, at high-pressure conditions, there was a slight difference of less than 0.01% in the calculated properties. This divergence is likely a result of employing different numerical methods to solve for the molar density.

The polytropic head and efficiency were verified by comparing them to the sample cases tested and calculated by (Huntington 2017). The analysis revealed that the maximum deviation between the two sets of results was less than 0.05%. This variance can be attributed to the utilization of different models for calculating the thermodynamic properties of the gas. Figures 2 and 3 depict graphical illustrations showcasing the calculated polytropic head and efficiency of a compressor employed in the injection process of sour gas, utilizing data collected over a period of one year. These figures correspond to the operational parameters specified in Table 4.

Table 4—Operating Parameters of the Examined Compressor

Parameter	Operating Range
Suction Pressure	14.5 MPa to 22 MPa
Suction Temperature	60 C° to 65C°
Discharge Pressure	38.7 MPa to 49.3 MPa
Discharge Temperature	125 C° to 145 C°

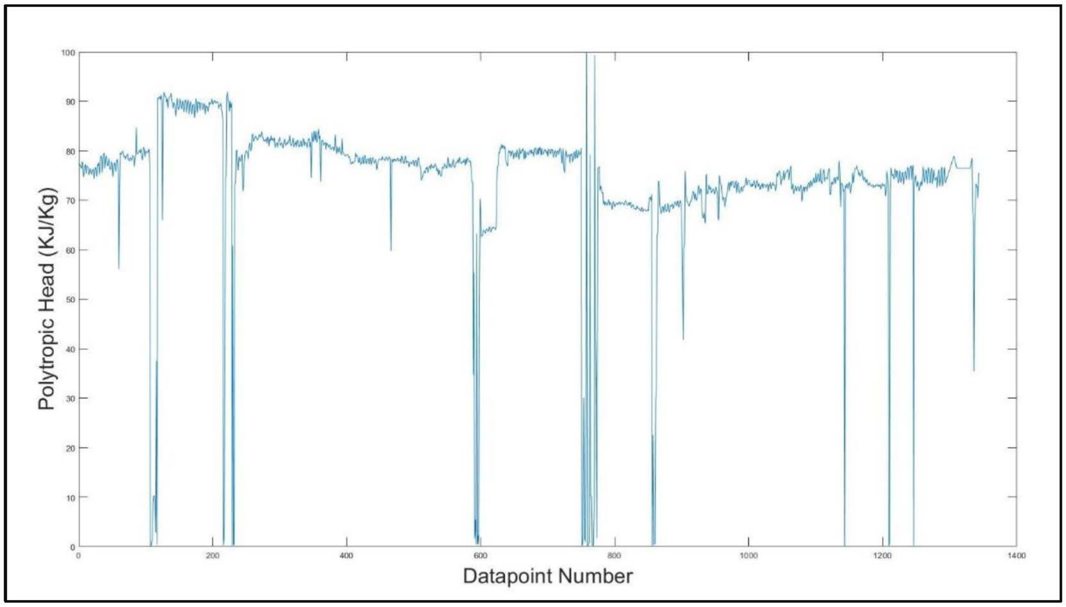

Figure 2—Calculated Polytropic Head using Huntington 4-Pont Method and GERG-2008 EoS

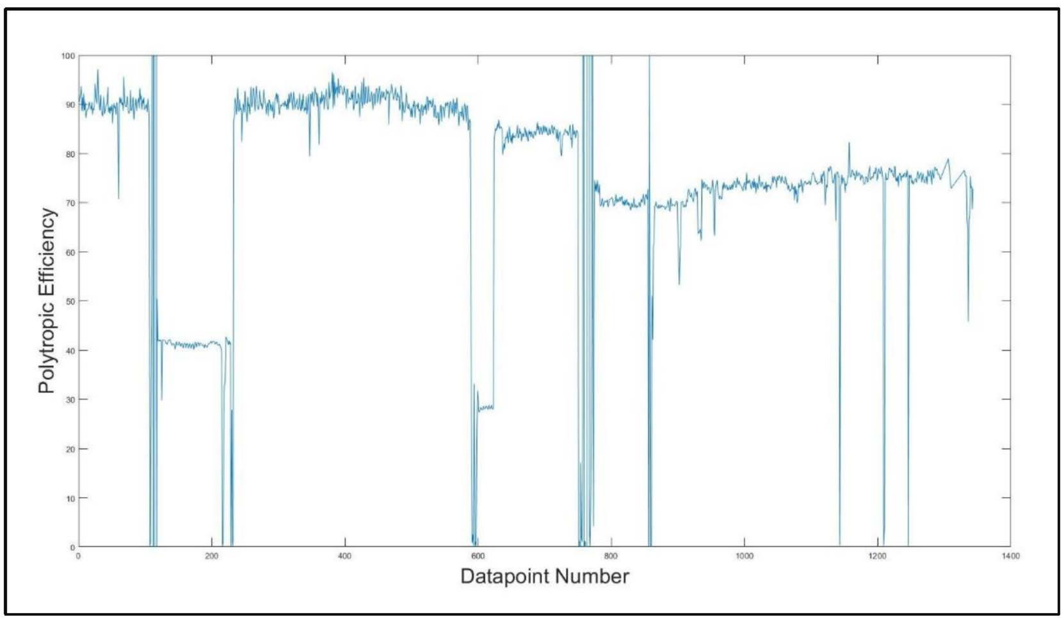

Figure 3—Calculated Polytropic Efficiency using Huntington 4-Pont Method and GERG-2008 EoS

The corrected performance maps at the operating suction conditions are derived by employing a Similitude model. In this study, a combination of two-dimensional linear interpolation and regression was used to work out the curves of the new Mach numbers. The linear model demonstrated an accuracy level of less than 5% deviation from the true values. Nevertheless, it is possible to enhance the accuracy to an error of 3% by augmenting the number of digitized points, particularly in the range characterized by high nonlinearity. Figure 4 showcases an example where the linearly interpolated and fitted non-dimensional map is superimposed onto the reference map. The reference map is depicted by the dotted curves, whereas the solid curves represent the derived map at the operating conditions.

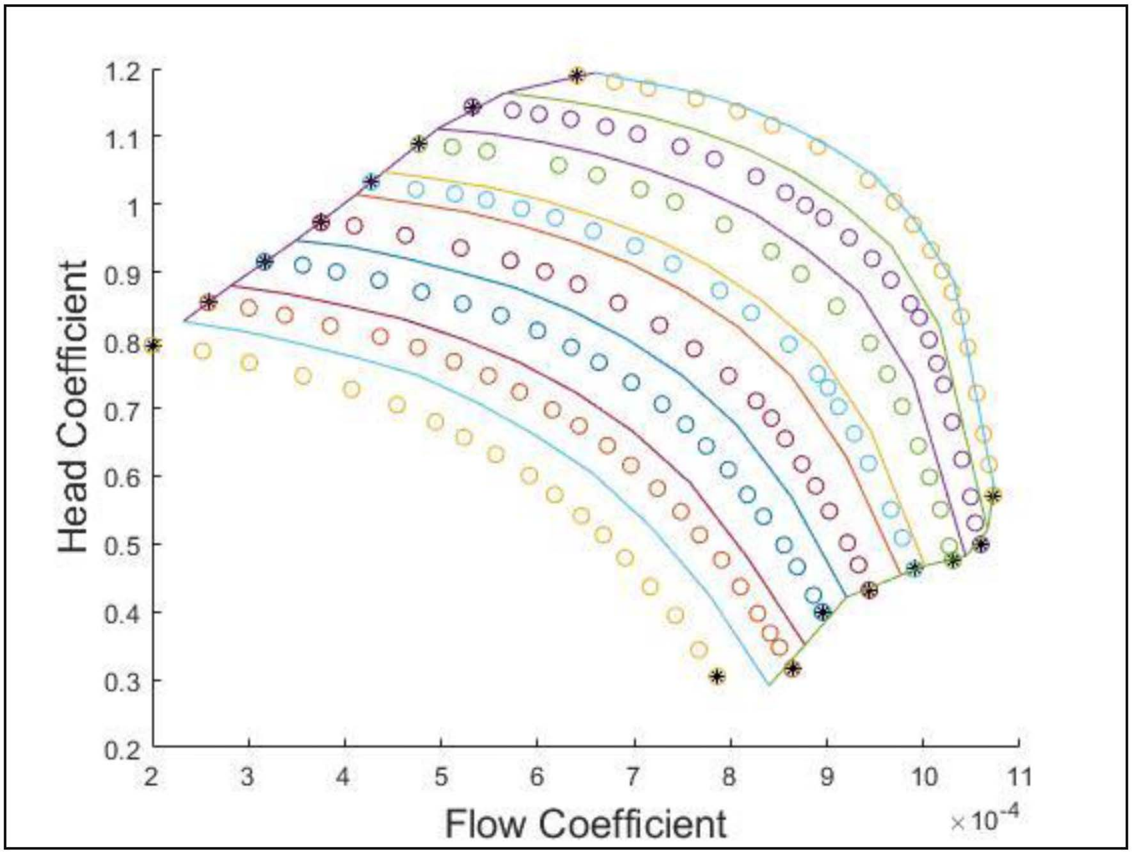

Figure 4—Non-dimensional Performance Map

Conclusion

The paper introduced a systematic approach aimed at evaluating the aerothermodynamic performance of centrifugal compressors. This approach is highly valuable for compressor end-users as it empowers them to conduct precise analyses. By utilizing this approach, compressor end-users can enhance maintenance optimization, decrease energy consumption, and improve cost-effectiveness. The validation of the GERG-2008 model involved comparing it with NIST REFPROP, while the Huntington 4-Point model was evaluated against results found in the literature. By using linear interpolation and regression to derive the curves at the new Mach numbers, the similitude model was examined and yielded satisfactory outcomes. The authors are presently exploring alternative interpolation and regression techniques, such as Lagrange and Spline methods. The outcomes will be published in a forthcoming publication.

Nomenclature

D	Impeller diameter
h	Enthalpy
M	Molar Mass
Mu	Mach number
n	Polytropic exponent
P	Pressure
Q	Volume flow rate
R	Gas constant
s	Entropy
T	Temperature
U	Impeller tip speed

v Molar volume
w Speed of sound
x Mole fraction
y Head/work
Z Compressibility factor
α Reduced Helmholtz free energy
γ Ratio of heat capacities
δ Reduced density
η Efficiency
ρ Density
τ Inverse reduced temperature
ϕ Flow coefficient
ψ Head coefficient
ω Acentric factor

Subscripts

c Critical
s Suction
R Reference conditions
d Discharge
p Polytropic
v volume

Superscripts

r Residual part/reference fluid
o Ideal gas
0 Simple fluid

References

Al Busaidi, Waleed, and Pericles Pilidis. 2016. "Investigation of Efficiency Deterioration Causes in Process Centrifugal Compressor Operation." *Journal of Failure Analysis and Prevention* **16**: 19–36.

1997. "ASME PTC-10." *Performance Test Code on Compressors and Expanders*. New York: American Society of Mechanical Engineers.

Aungier, Ronald H. 2000. Centrifugal Compressors: A Strategy for Aerodynamic Design and Analysis. New York: ASME Press.

Benedict, Manson, G. B. Webb, Rubin, and C. Louis. 1940. "An Empirical Equation for Thermodynamic Properties of Light Hydrocarbons and Their Mixtures i. Methane, Ethane, Propane and n-Butane." *Journal of Chemical Physics* **8**: 334–345.

Evans, Fred, and Spencer Huble. 2017. "Centrifugal Compressor Performance Making Enlightened Analysis Decisions." *46th Turbomachinery and 33rd Pump Symposia*. Houston: George R. Brown Convention Center.

Huntington, Richard A. 2017. Another New Look at Polytropic Calculations Methods for Turbomachinery. Technical Report, Research Gate.

Kowng, Otto, and Josephine Redlich. 1949. "On the Thermodynamics of Solutions; an Equation of State; Fugacities of Gaseous Solutions." *Chemical Reviews* **44-1**: 233–244.

Kunz, O., R. Klimeck, W. Wagner, and M. Jaeschke. 2007. "The GERG-2004 Wide-Range Equation of State for Natural Gases and Other Mixtures." Düsseldorf: VDI Verlag GmbH.

Lee, Byung I.K., and Michael G. Kesler. 1975. "A Generalized Thermodynamic Correlation Based on Three-Parameter Corresponding States." *AIChE Journal* **21**(3): 510–527.

Lüdtke, Klaus. 2004. Process Centrifugal Compressors. Berlin: Springer-Verlag.

Peng, D.Y., and D.B. Robinson. 1976. "A New Two-Constant Equation of State." *Industrial and Engineering Chemistry Fundamental* **15**: 59–64.

Plöcker, U. 1977. "Doctoral Thesis." Berlin: Technical University of Berlin.

Robinson, Chris, and Michael Casey. 2021. "Radial Flow Turbocompressors: Design, Analysis, and Applications." Cambridge: Cambridge University Press.

Schultz, John M. 1962. "The Polytropic Analysis of Centrifugal Compressors." *Journal of Engineering for Power* **84**: 69–82.

Starling, and Savidge. 1992. Compressibility Factors of Natural Gas and Other Related Hydrocarbon Gases. American Gas Association, 205.

Taher, Matt, and Fred Evans. 2021. "Centrifugal Compressor Polytropic Performance—Improved Rapid Calculation Results—Cubic Polynomial Methods." *International Journal of Turbomachinery Propulsion and Power*.

van der Waals, Johannes Diderik. 1873. "de Continuiteit van den Gas- en Vloeistoftoestand2." Leiden: University of Leiden.

Wagner, W., and O. Kunz. 2012. "The GERG-2008 Wide-Range Equation of State for Natural Gases and Other Mixtures: An Expansion of GERG-2004." *Journal of Chemical Engineering Data* **57**: 3032–3091.